Practice 1_Suggested solution (you might have different solution that are also acceptable)

1. Solve the following equations.

$$\frac{x-4}{3} - 5 = 0$$

$$\frac{x-4}{3} = 0 + 5$$

$$x - 4 = 5 \times 3$$

$$x = 15 + 4$$

$$x = 19$$

$$\frac{x-6}{5} - 7 = 0$$

$$\frac{x-6}{5} = 0 + 7$$

$$x - 6 = 7 \times 5$$

$$x = 35 + 6$$

$$x = 41$$

$$\frac{x-9}{8} - 10 = 0$$

$$\frac{x-9}{8} = 0 + 10$$

$$x - 9 = 10 \times 8$$

$$x = 80 + 9$$

$$x = 89$$

2. Explore the pattern hidden in the equations.

In the above equations, the three numbers on the left hand side of the equation follow the same pattern and they are always consecutive numbers [a – 1, a and a + 1].

The unknown (x) on the numerator minus a certain number (a) [i.e. x – a] divide by the number minus one [a – 1] as in the denominator, then the quotient minus the certain number plus one [a+1] will always equal to 0.

The value of x is always a prime number.

The value of x can be calculated by multiplying the biggest and smallest number then add the middle number [$x = (a - 1) \times (a+1) + a$].

3. Express this pattern as a general rule.

a. Let the certain number be n, then the general rule of the equation can be written as the following equation:

$$\frac{x-n}{n-1} - (n+1) = 0$$

b. If the equation is rewritten as follow:

$$\frac{x-a}{b} - c = 0$$

then, b = a – 1 and c = a+1

c. The solution x can be found by the following equation

i. x = c x b + a or

ii. x = (a + 1)(a – 1) + a

4. Demonstrate that your general rule works for any case of the pattern.

To demonstrate my general rule works for any case of the pattern, I try to make up two more equations that fulfill the pattern I found and see if they works or not.

a) If I let the certain number be 10, the equation becomes:

$$\frac{x-10}{9} - 11 = 0$$

$$\frac{x-10}{9} = 0 + 11$$

$$x - 10 = 11 \times 9$$

$$x = 99 + 10$$

$$x = 109$$

Checking: (when x = 109)

$$\text{L.H.S.} = \frac{x-10}{9} - 11$$
$$= \frac{109-10}{9} - 11$$
$$= \frac{99}{9} - 11$$
$$= 11 - 11$$
$$= 0$$
$$= \text{R.H.S.}$$

Now, I solve this equation with my general rule:

x = (a + 1)(a – 1) + a

x = (10 + 1)(10 – 1) + 10

x = 11 × 9 + 10

x = 99 + 10

x = 109

x is a prime number and the solution is the same as the answer get by solving equation.

b) This time I will try to use 3, a prime number as the certain number a, the equation becomes:

$$\frac{x-3}{2} - 4 = 0$$

$$\frac{x-3}{2} = 0 + 4$$

$$x - 3 = 4 \times 2$$

$$x = 8 + 3$$

$$x = 11$$

Checking: (when x = 11)

$$\text{L.H.S.} = \frac{x-3}{2} - 4$$
$$= \frac{11-3}{2} - 4$$
$$= \frac{8}{2} - 4$$
$$= 4 - 4$$
$$= 0$$
$$= \text{R.H.S.}$$

Now, I solve this equation with my general rule:

$$x = (a + 1)(a - 1) + a$$

$$x = (3 + 1)(3 - 1) + 3$$

$$x = 4 \times 2 + 3$$

$$x = 8 + 3$$

$$x = 11$$

x is a prime number and the solution is the same as the answer get by solving equation.

After trying for 2 cases with different type of numbers (composite and prime), I found out that they all fulfill the general rules so I have demonstrated that my general rules work for any case of the pattern.